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A holographic dual of CFT with flavor on de Sitter space

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Abstract: We introduce a D7-brane probe in $AdS_5 \times S^5$ background in a way that the 4d part of the induced metric on D7-brane becomes 4d de-Sitter space (dS_4) inside AdS_5 instead of 4d Minkowski space. Although supersymmetry is completely broken, we obtain a static configuration and show the absence of dynamical tachyonic modes. Following holographic renormalization we renormalize the Dirac-Born-Infeld action of D7-brane and we completely fix the counter terms including finite contributions from the consistency under various limits. Through the AdS/CFT correspondence we study the chiral condensate and meson spectrum of CFT dual theory on dS_4 where the energy scale is identified with the direction normal to dS_4 space in AdS_5. We identify and properly reproduce the finite temperature effects on dS_4. Our results support the holographic interpretation of the Randall-Sundrum model with non fine-tuned dS_4 brane(s) and the holography between AdS_p (or dS_p) bulk gravity and CFT on dS_{p-1} called the (A)dS/dS correspondence.

Keywords: Brane Dynamics in Gauge Theories, dS vacua in string theory, AdS-CFT Correspondence, Thermal Field Theory.

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1. Introduction

The holographic principle conjectured by 't Hooft [1] and Susskind [2] is one of the most important keys to understand and construct quantum theory of gravity. This proposal deeply concerns the fundamental principles of spacetime and quantum theory. The explicit example of holography was realized in string theory with D-branes by Maldacena [3] and is called the AdS/CFT correspondence. The best understood example of the correspondence is supergravity (typeIIB string theory) on $AdS_5 \times S^5$ is dual to 4d N=4 super Yang-Mills theory (SYM). A field on AdS is identified with an operator in CFT and the generating functional of correlators in CFT is calculated from AdS gravity [4].

Many deformations of $\mathrm{AdS}_5/\mathrm{CFT}_4$ have been discussed by compactifying one or more spacetime directions, replacing AdS by asymptotically AdS geometries and so on, and the parameters describing modifications correspond to the temperature or coupling constants which may break some of supersymmetry in CFT side (see [5] and also [6] for a review).

The holographic principle has been also conjectured for other spacetime backgrounds which have not been yet realized or well understood in string theory. Strominger discusses the dS/CFT correspondence [7], and a Minkowski/CFT correspondence is also proposed [8]. The holographic interpretation of the Randall-Sundrum model [9] was developed in [10]. Hawking-Maldacena-Strominger [11] speculate the AdS/CFT interpretation of the Randall-Sundrum model with non fine-tuned branes, i.e. AdS_5 space bounded by dS_4 branes, by deriving the same value of the entropy from AdS gravity and from CFT on a dS space. Alishahiha et al. [12, 13] discuss a broader holography between dS_p or AdS_p bulk gravity and CFT on dS_{p-1} which is realized as a slice of bulk geometry, the (A)dS/dS correspondence (see also [14] for other attempts to realize curved spaces).

However since the explicit string realization of these spacetime backgrounds has not been well understood, many of these arguments are limited to be qualitative. Moreover in the last two examples, the holography is in some sense unexpected since the same AdS geometry has different CFT dual descriptions depending on how AdS geometry is sliced, a Minkowski slice or a de-Sitter slice.

A classical configuration of a bulk field in AdS_5 contains the information of correlation function of the corresponding operator in CFT. Let us discuss a static configuration along 4d Minkowski space. The configuration is thus a function of normal direction to 4d Minkowski space. There are two independent integration constants in the asymptotic solutions near the AdS boundary. They correspond to the coupling constant and the vacuum expectation value of the corresponding CFT operator. Those are, of course, some numerical numbers. Now let us apply the same logic to a static configuration along dS_4 space in the (A)dS/dS correspondence. (In this paper we only consider AdS_p bulk and do not consider dS_p bulk geometry.) The static configuration is a function of normal direction to dS_4 which is NOT a normal direction to 4d Minkowski space, but a combination of that and time direction of 4d Minkowski space. Using the original AdS/CFT correspondence, we obtain a time dependent coupling constant and a time dependent vacuum expectation value of the same CFT operator (section 5). Although we are studying the same theory, we are looking at a sector where 4d Lorentz symmetry is complicatedly broken. So what is this sector? The guide is the symmetry. Since the static configuration is independent of dS_4 coordinates, the coupling constant and vacuum expectation value are interpreted as those on dS_4. We will explain this idea in section 5 and speculate a conformal transformation relates the (A)dS/dS correspondence (AdS_p bulk) with the AdS/CFT correspondence.

Without string realization, the above argument just tells a possibility that a different slice of AdS gives a different result on the dual theory. In this paper we realize the setup of the (A)dS/dS correspondence in string theory in a probe limit. The gravity dual of adding small number of dynamical quarks into N=4 SYM has been proposed by Karch and Katz by introducing a D7-brane as a probe in the D3-brane background $AdS_5 \times S^5$ [15]. We introduce a probe D7-brane in a different way such that the probe D7-brane fills dS_4 inside AdS_5 and other directions except the S^2 inside the S^5 (section 2). The locus in the S^2 is described by a scalar field localized on D7-brane and is a function of the normal direction to dS_4 surface in AdS_5. In the CFT side this corresponds to introducing fundamental quarks with supersymmetry breaking mass terms (see section 5). We find a stable supergravity configuration and this realizes the setup of the (A)dS/dS correspondence, since the scalar field is a bulk field in AdS_5 and is a function of the direction normal to dS_4. Then applying the AdS/CFT dictionary [4] and holographic renormalization [16, 17], we show the corresponding CFT operator is the quark anti-quark composite operator as it is in [15] and study the flavor physics which are the chiral condensate (section 2), meson spectrum (section 3) and quark anti-quark potential (section 4). We then discuss whether the flavor physics we obtain is consistently understood as that in CFT with flavor on dS_4.

The dS_4 space has a temperature proportional to the inverse of curvature radius. We expect the finite temperature effects on flavor physics do not severely depend on the detail of large N gauge theory, and are qualitatively same as those in a large N gauge theory

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with flavor on Minkowski space at finite temperature. In fact many works on flavor physics using the gravity dual of nonsupersymmetric large N gauge theories [18, 19] have shown that the qualitative behaviour of flavor physics does not depend on the detail of large N gauge theory. It has been also known that the finite temperature effects on flavor physics are insensitive to the detail of large N gauge theory [20, 21, 18, 22].

Therefore we will thus identify the finite temperature effects on flavor physics in our calculations and see whether they are properly reproduced. We will show that they are indeed properly reproduced. Thus we believe our results give an support on the holographic understanding of the Randall-Sundrum model with non fine-tuned dS_4 brane(s) and the (A)dS/dS correspondence.

2. Probe D7-brane and the chiral condensate

The effective theory on the supersymmetric D3/D7 system is known to be N=2 SYM with charged hypermultiplets which are incorporated from the D3-D7 open strings. Karch and Katz proposed the corresponding supergravity solution by treating D7-brane as a probe[1] in $AdS_5 \times S^5$ geometry [15]. In this paper we instead embed a D7-brane probe in a way that supersymmetry is completely broken. The AdS_5 space realizes a dS_4 space as a hypersurface in addition to 4d Minkowski space and we embed a probe D7-brane along this dS_4 space (thus supersymmetry is completely broken) and the normal direction to the dS_4 hypersurface in AdS_5 as well as the S^3 inside S^5. The position of D7-brane in the remaining S^2 directions inside S^5 is a function of the normal direction to dS_4. In the CFT side, introducing D7-brane corresponds to adding fundamental quarks and all the results we will compute suggest that we add fundamental quarks on dS_4 space.

The D3-brane background is described by $AdS_5 \times S^5$ (appendix A)

$$\begin{aligned} ds^2 &= \frac{R^2}{l_5^2} dx_{M4}^2 + \frac{l_5^2}{R^2} dR^2 + l_5^2 d\Omega_5^2 && (2.1) \\ &= \left(\frac{u^2}{l_5^2} - \frac{l_5^2}{l_4^2}\right) dx_{dS4}^2 + \left(\frac{u^2}{l_5^2} - \frac{l_5^2}{l_4^2}\right)^{-1} du^2 + l_5^2 d\Omega_5^2 && (2.2) \end{aligned}$$

where 4d Minkowski space (4d de-Sitter space) is apparent in the first (second) expression. $d\Omega_n^2$ is the metric of S^n. We explicitly write the AdS_5 and dS_4 curvature radius which are l_5 and l_4 respectively. R is the distance from D3-brane, $R^2 = (X^4)^2 + \cdots (X^9)^2$ where $X^0 \cdots X^9$ are the rectangular coordinates. As approaching to the AdS_5 boundary, $u \to \infty$, these two expressions are same at the leading order and this is understood that the dS_4 curvature effects are ignored in high energy limit in the gauge theory side.

Now we embed a dS_4 space filling D7-brane as a probe in this geometry. For the computational convenience, we change the coordinate u into r by

$$\frac{4r}{l_5} = \left(\sqrt{\frac{u}{l_5} + \frac{l_5}{l_4}} + \sqrt{\frac{u}{l_5} - \frac{l_5}{l_4}}\right)^2 \tag{2.3}$$

[1]The backreaction of a probe is discussed in [23].

and the $AdS_5 \times S^5$ metric becomes

$$ds^2 = r^2 \left(\frac{1}{l_5} - \frac{l_5^3}{4r^2 l_4^2} \right)^2 dx_{dS4}^2 + \frac{l_5^2}{r^2}(dr^2 + r^2 d\Omega_5^2). \tag{2.4}$$

We further rewrite $dr^2 + r^2 d\Omega_5^2 = d\rho^2 + \rho^2 d\Omega_3^2 + dy^2 + dz^2$ and then

$$ds^2 = r^2 \left(\frac{1}{l_5} - \frac{l_5^3}{4r^2 l_4^2} \right)^2 dx_{dS4}^2 + \frac{l_5^2}{r^2}(d\rho^2 + \rho^2 d\Omega_3^2 + dy^2 + dz^2), \qquad r^2 = \rho^2 + y^2 + z^2 \, . \tag{2.5}$$

In this coordinate system, the world volume of dS_4 space filling D7-brane is embedded with $y = y(\rho)$ and $z = 0$. Because of the rotational symmetry, we place the brane at $z = 0$ and the position along y becomes a non trivial function of ρ.

Since D7-brane breaks supersymmetry completely, this static configuration may have tachyonic directions[2] and we should include the time dependence. However as long as the energy scale associated with the motion of D7-brane (the scale of tachyon condensation) is much smaller than the scales which we are interested in, such as l_5^{-1}, l_4^{-1} and the distance between D3 and D7 branes which will be understood as the mass for charged matter fields, we expect that the AdS/CFT correspondence is still applicable and the instability of D7-brane embedding should be interpreted as an instability in the field theory side. (Actually we will show there are no dynamical tachyonic modes.)

The Dirac-Born-Infield action of D7-brane with the tension T_7 and the equation of motion for $y(\rho)$ are

$$S = -T_7 \int d^8x \sqrt{-h} \propto \int d\rho \left(1 - \frac{1}{4r^2 l_4^2} \right)^4 \rho^3 \sqrt{1 + y'(\rho)^2}, \tag{2.6}$$

$$y''(\rho) = -3 \frac{(1 + y'(\rho)^2)}{\rho} y'(\rho) + \frac{2}{l_4^2 r^2 - \frac{1}{4}} \frac{y(\rho) - \rho y'(\rho)}{r^2} (1 + y'(\rho)^2) \tag{2.7}$$

where $\sqrt{-h}$ is the determinant of metric and we hereafter use $l_5 = 1$ unit. The D7-brane configuration is given by a regular solution of the equation of motion up to $\rho = 0$ or up to the horizon, otherwise the approximation of using supergravity breaks down and there is no interpretation of gauge theory dual since r direction is related with the energy scale. When we separate one D3 brane from others along r direction, the mass M_W of gauge boson associated with symmetry breaking is calculated from the tension of string stretched between the probe D3 brane and the horizon [12],

$$M_W \propto \frac{1}{\alpha'} \int_{r_h}^{r} dr \sqrt{g_{00}} \sqrt{g_{rr}} = \frac{1}{\alpha'} \left(r - r_h + \frac{1}{4r l_4^2} - \frac{1}{4r_h l_4^2} \right) \tag{2.8}$$

where r and r_h are the position of probe D3 brane and horizon in (2.4) and $1/\alpha'$ is the string tension. Therefore the proper distance from the horizon along r direction is related with the energy scale in the dual field theory [12].

[2]Some of readers may wonder if there is a static embedding. Since the regularity of $y(\rho)$ gives only one condition for a second order differential equation for $y(\rho)$, we always have a static solution.

In the AdS/CFT correspondence the correlation functions of CFT operators are computed from the on-shell supergravity action which is obtained from plugging a solution. The on-shell action is usually divergent and we should regularize and renormalize the action to have a well defined action. Therefore we first study asymptotic solutions and divergences in the on-shell action. The asymptotic solutions at $\rho \to \infty$ are

$$y(\rho) = m\left(1 - \frac{\ln(\rho^2 + m^2)}{2l_4^2\rho^2}\right) + \frac{v}{\rho^2} + O(\rho^{-4}). \tag{2.9}$$

Though v and m are integration constants in this order, v is determined in terms of m to give a regular solution, i.e. $y(\rho)$ and $y'(\rho)$ are everywhere finite. Plugging the asymptotic solution into the Dirac-Born-Infield action, we obtain

$$S = -T_7\Omega_3 \int d^4x\sqrt{-g_{dS4}}\left\{\frac{\rho_\infty^4}{4} - \frac{\rho_\infty^2}{2l_4^2} + \left(\frac{3}{8l_4^4} + \frac{m^2}{l_4^2}\right)\ln\rho_\infty\right\} + S_f \tag{2.10}$$

where Ω_3 is the volume of S^3 and g_{dS4} is the dS_4 metric. We regularize the action by introducing the cutoff ρ_∞, which will be taken infinity after renormalization and S_f is the finite part. We notice that when m is smaller than roughly $1/l_4$ ($1.4/l_4$ from numerical analysis, see some regular solutions in figure 1), a regular solution y ends at the horizon, and it is easily checked that there is no divergent contributions to the on-shell action from the horizon. The coefficient in front of $\ln\rho_\infty$ is understood as the conformal anomaly of CFT [16] and is same as that calculated in [13]. Since the m dependent term in the potential is unbounded below, the mode associated with changing m is a 'tachyonic' mode. However the change of potential energy is (log) divergent and thus this 'tachyonic' mode is not a dynamical mode (an infinite energy is needed to turn on this mode). After we introduce counter terms and renormalize the action, the finite on-shell action would depend on m. However we can impose the Dirichlet type boundary condition for $y(\rho_\infty)$ as is done in AdS/CFT and fix m, i.e. we can freeze the instability mode. We will later study there is no tachyonic mode in local excitations. Thus we apply the AdS/CFT correspondence up to the low energy limit in the field theory side.

Although in this embedding introducing the cutoff at constant ρ seems natural, this is the key. We do not introduce the cutoff at constant R in the metric (2.1). We just simply follow the AdS/CFT dictionary [4] and holographic renormalization [16, 17] with replacing R by ρ, and compute correlation functions in the dual theory. Then we will check whether the resultant correlation functions are consistently understood as those in CFT with flavor on dS_4.

Since the symmetry of action is useful for determining the possible counter terms, we notice the invariance of metric under the following redefinition parametrized by an arbitrary real parameter α

$$l_4 \to \alpha l_4, \quad \rho \to \alpha^{-1}\rho, \quad y \to \alpha^{-1}y. \tag{2.11}$$

Therefore although we have two parameters l_4 and m in the asymptotic solutions (2.9) after taking $l_5 = 1$ unit, the real free parameter is only the combination $ml_4 \propto m/T$ where T is the temperature of dS_4 space. However we treat both l_4 and m as parameters since it is convenient for discussing the behaviour under the change of l_4 and m.

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Following the holographic renormalization [16, 17], we introduce the counter terms which should respect unbroken symmetries and are defined at the cutoff $\rho = \rho_\infty$. Those are

$$
\begin{aligned}
S_c &= T_7\Omega_3 \int d^4x\sqrt{-\gamma}\, F\left(\frac{y^2}{\rho^2}, \rho^2 l_4^2\right)\bigg|_{\rho=\rho_\infty, y=y(\rho_\infty)}\,, && (2.12)\\
\sqrt{-\gamma} &= \sqrt{-g_{dS4}}\, r^4\left(1-\frac{1}{4r^2l_4^2}\right)^4 && (2.13)
\end{aligned}
$$

where γ is the induced metric at a constant $\rho = \rho_\infty$ surface in AdS_5 and y appears in the combination y/ρ since y/ρ is the canonically normalized field in AdS_5 and invariant under (2.11), and so is ρl_4. The form of F is determined to cancel the divergences in S and we obtain

$$
\begin{aligned}
S_c = T_7\Omega_3 \int d^4x\sqrt{-\gamma}\bigg[&\frac{1}{4}-\frac{y^2}{2\rho^2}-\frac{1}{4\rho^2 l_4^2}+\left(\frac{3}{8l_4^4\rho^4}+\frac{y^2}{l_4^2\rho^4}\right)\left\{(c_1+1)\ln(l_4\rho)+c_1\ln\left(\frac{y}{\rho}\right)\right\}\\
&+c_2\frac{y^4}{\rho^4}+c_3\frac{1}{\rho^4 l_4^4}+c_4\frac{y^2}{\rho^4 l_4^2}\bigg]\bigg|_{\rho=\rho_\infty, y=y(\rho_\infty)}\,. \qquad (2.14)
\end{aligned}
$$

With these coefficients, all the divergences are cancelled out, i.e. $S + S_c$ is finite. As is discussed in the literatures [16, 17, 24], we have ambiguities in finite contributions $c_1 \cdots c_4$ which are numerical constants and do not depend on l_4. A different choice of finite counter terms corresponds to a different regularization scheme in the field theory side and the correlation functions depend on a choice of scheme. Thus we need to find a proper regularization scheme. We will show that the finite counter terms are completely fixed from the requirements that the correlation functions should recover those in supersymmetric theories after taking the limit $l_4 \to \infty$ and should also recover physically acceptable ones in the limit where dS_4 curvature is negligible.

Since the metric (2.2) approaches to the metric (2.1) near the AdS boundary, we identify the CFT operator $\mathcal{O}$ for y field as the same operator in the supersymmetric case, i.e. the quark anti-quark composite operator $q\bar{q}$ where quark is a fermionic field in a hypermultiplet. According to the AdS/CFT dictionary [4], $y(\rho_\infty \to \infty) = m$ is the mass for quark, i.e. $\mathcal{L} = \mathcal{L}_{CFT} + m\bar{q}q$, and the one point function $\langle\mathcal{O}\rangle$, the chiral condensate, is calculated from the renormalized action,

$$
\begin{aligned}
\langle\mathcal{O}\rangle &= \lim_{\rho_\infty\to\infty}\frac{\rho_\infty^4}{\sqrt{-\gamma}}\frac{\delta(S+S_c)}{\delta y(\rho_\infty)} && (2.15)\\
&= \lim_{\rho_\infty\to\infty} T_7\Omega_3\left[\left\{-\frac{2m}{l_4^2}\ln\rho_\infty+\frac{m}{l_4^2}+2v\right\}+\left\{\frac{2m}{l_4^2}\ln\rho_\infty+\frac{m}{l_4^2}(A+B\ln m)+Cm^3\right\}\right],\\
A &= 2(c_1+1)\ln l_4 + c_1 + 2c_4, \qquad B = 2c_1, \qquad C = 4c_2\,, && (2.16)
\end{aligned}
$$

where γ is the induced metric at a constant $\rho = \rho_\infty$ as (2.13). The terms inside the first bracket come from the action S and those inside the second bracket come from the counter terms S_c. The log divergent terms cancel out and finite counter terms do contribute to the one point function. In the supersymmetric case, i.e. $l_4 \to \infty$, $2v + Cm^3$ survive. Since the

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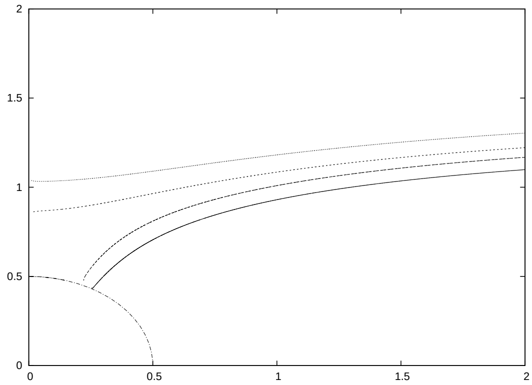

Figure 1: The regular solutions $y(\rho)$ vs ρ. From the top $m = 1.5, 1.43, 1.37$ and 1.3. The horizon radius is 0.5. $l_4 = 1$.

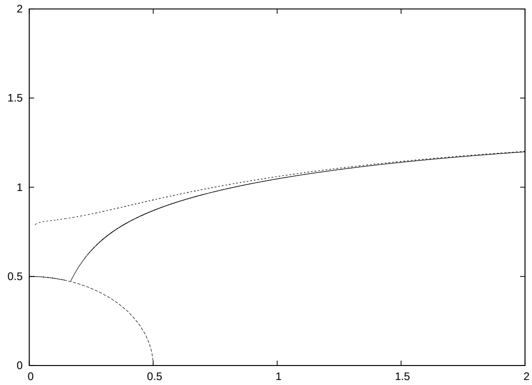

Figure 2: The multiple solutions $y(\rho)$ at $m = 1.4$ vs ρ. $l_4 = 1$.

regularity of solution is satisfied only for $v = 0$ and the total on-shell value of action should be zero because of supersymmetry [24], $C = 0$ follows and there is no condensate. When l_4 is finite, v in a regular solution is a complicated function of m. We expect the chiral condensate $\langle \mathcal{O} \rangle$ goes to zero as quark mass gets large $m \gg 1/l_4$ because the dS_4 curvature effects should be negligible, and thus other coefficients A and B should be determined such that $\langle \mathcal{O} \rangle$ goes to zero as $ml_4 \to \infty$. If we cannot realize $\langle \mathcal{O} \rangle \to 0$ in this limit, it indicates the AdS/CFT correspondence has to be modified or cannot apply for this case. Therefore this gives a nontrivial check of the AdS/CFT correspondence in our generalized cases.

The ambiguities of finite counter terms exist in the case of other geometries as well. For example in the case of AdS Schwartzschild black hole studied in [21], there is one free parameter labells the finite counter term which corresponds to C in (2.16). This free parameter is fixed such that the chiral condensate goes to zero at the supersymmetric limit or a large quark mass $m/T \to \infty$ (T is the black hole temperature) limit and $C = 0$ is obtained. Thus the chiral condensate in [21] is just given by v in our notation.

We solve the equation of motion numerically and find regular solutions of y by the shooting method. We set $l_4 = 1$ and give the initial conditions read from (2.9) at $\rho^2 = 5000$ and solve the equation of motion toward $\rho = 0$ with changing v and m to find regular solutions. By using the rescaling (2.11), a set of regular solution (m,v) for $l_4 = 1$ gives another one for l_4 with (m/l_4, $v/l_4^3 - m \ln l_4/l_4^3$). When $m \lesssim 1.4/l_4$, we find the regular solution ends at the horizon (figure 1). We show the numerical results of the sets of (v, m) which give regular solutions denoted by dots in figure 3 for $l_4 = l_5 = 1$ case. v is an increasing function of m and it is nontrivial if finite counter terms can cancel the increasing behaviour of v. The numerical data is best fit[3] with $1.0 \times m \ln m$ which is shown in figure 3 as a line, and then we obtain $c_1 = -1$, $c_2 = 0$ and $c_4 = 0$ in our parameters (2.14). (The parameter c_3 would be fixed similarly from minimising the on-shell action.) Although the line in figure 3 fits the data very well, the line does not completely fit them and the small difference between the line and the dotted data becomes

[3]We obtain the fit using the data at $m = 40$ and $m = 50$.

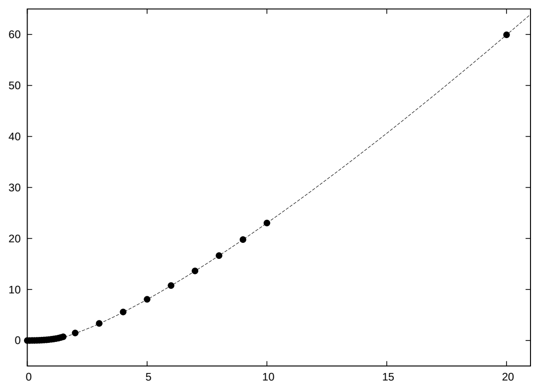

Figure 3: v of regular solutions vs m. The line is the best fit function $1.0 \times m \ln m$ which almost overwraps the dots in this scale. $l_4 = 1$.

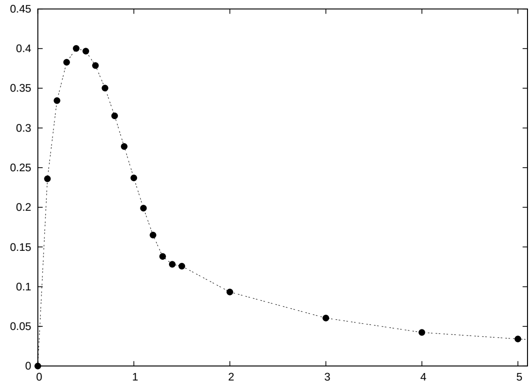

Figure 4: The chiral condensate $\langle \mathcal{O} \rangle$ vs m. $l_4 = 1$. (The small difference between the line and numerical data in figure 3 gives this.)

our prediction of chiral condensate which is shown in figure 4 with the unit $2T_7\Omega_3 = 1$. Since $y(\rho) = y'(\rho) = 0$ is a solution, there is no quark condensate in the chiral limit. The behaviour of chiral condensate is very similar to that in the large N gauge theory with flavor at finite temperature studied from AdS_5 Schwartzschild black hole [21]. Therefore we have properly picked up finite temperature effects in our D7-brane embedding. Some comments are in order: 1) When a D7-brane ends at the horizon, it is nontrivial whether the horizon is stable under perturbations. However since this is the AdS horizon, we expect the horizon is stable under perturbations. In fact we will study perturbations around the solution in the next section and show there are no tachyonic modes. 2) We find multiple solutions in the region $1.38 \leq ml_4 \leq 1.41$ (see figure 2) as are found in [18]. This might imply a phase transition at $m \sim 1.4/l_4$ as studied in QCD at the finite temperature [18]. 3) Although the final result in figure 4 is very similar to that in [21], the geometries in this paper and in [21] are different. Before introducing the probe D7 brane, the goemetry in this paper is just AdS_5 and the $N = 4$ supersymmetry is kept unbroken which is on the contrary broken in the case of AdS_5 black hole. Since we study the situation in which the probe approximation is a good approximation, the backreaction of D7 brane is small and the geometries are still different.

3. Meson spectrum

We have not shown there are no tachyonic modes in local excitations around a regular solution. In the gauge theory side, the fluctuations about a D7-brane configulation correspond to meson fields [25]. In this section we thus aim to study the meson mass spectrum and see if there are tachyonic modes and if the meson spectrum captures finite temperature effects.

We first discuss what are expected from the results in the previous section. Since there is no chiral symmetry breaking in the chiral limit, it is consistent that we have a mass gap in the spectrum contrary to QCD at low temperature. The spectrum of heavy masses, much heavier than $1/l_4$, should be approaching to the mass spectrum in supersymmetric case plus thermal corrections. With keeping these in mind, we now solve linearlized equations of motion. The small (s-mode) fluctuations along z direction[4] are included in the induced metric

$$ds^2 = r^2\left(1 - \frac{1}{4r^2 l_4^2}\right)^2 dx_{dS4}^2 + \frac{1}{r^2}\left(d\rho^2 + \rho^2 d\Omega_3 + y'(\rho)^2 d\rho^2 + \partial_a z \partial_b z dx^a dx^b\right) \tag{3.1}$$

and the action up to the quadratic terms in terms of z becomes

$$\begin{aligned} S = -T_7\Omega_3 \int d^4x d\rho \left(1 - \frac{1}{4r_0^2 l_4^2}\right)^4 \rho^3 \sqrt{1 + y'(\rho)^2} \Bigg\{ 1 + \left(1 - \frac{1}{4r_0^2 l_4^2}\right)^{-2} \frac{g_{dS4}^{\mu\nu}\partial_\mu z \partial_\nu z}{2r_0^4} \\ + \left(1 - \frac{1}{4r_0^2 l_4^2}\right)^{-1} \frac{z^2}{r_0^4 l_4^2} + \frac{(\partial_\rho z)^2}{2(1 + y'(\rho)^2)} \Bigg\} \end{aligned} \tag{3.2}$$

where $r_0^2 = \rho^2 + y(\rho)^2$ and $y(\rho)$ is the regular solution obtained in the previous section. Then it is easily seen that the energy carried by z (s-mode) fluctuations is positive definite and we do not have tachyonic modes. Notice that the counter terms S_c do not play any rules in this discussion. Since our main object in this section is the meson spectrum, we proceed to solve the equations of motion for z.

The linearized equation of motion for a Kaluza-Klein (KK) mode $z = \phi_M(x) f(\rho)$ with the four dimensional mass M becomes

$$\begin{aligned} \partial_\rho^2 f(\rho) = \left\{ -\frac{3}{\rho} + \frac{y'(\rho) y''(\rho)}{1 + y'^2(\rho)} - \frac{2(\rho + y(\rho) y'(\rho))}{r_0^4 l_4^2} \left(1 - \frac{1}{4r_0^2 l_4^2}\right)^{-1} \right\} \partial_\rho f(\rho) \\ + (1 + y'(\rho)^2) \left(1 - \frac{1}{4r_0^2 l_4^2}\right)^{-1} \left\{ \frac{2}{r_0^4 l_4^2} - \left(1 - \frac{1}{4r_0^2 l_4^2}\right)^{-1} \frac{M^2}{r_0^4} \right\} f(\rho). \end{aligned} \tag{3.3}$$

After normalizing properly, this Kaluza-Klein mode then satisfies

$$S = -\int d^4x \sqrt{-g_{dS4}} (g_{dS4}^{\mu\nu} \partial_\mu \phi_M \partial_\nu \phi_M + M^2 \phi_M^2). \tag{3.4}$$

The asymptotic solutions near the AdS boundary are same as those for y and so $f(\rho)$ should be proportional to $1/\rho^2$ near the boundary. We give this boundary condition at $\rho^2 = 5000$ and solve the equation numerically toward $\rho = 0$. Since $f(\rho_\infty) \propto 1/\rho_\infty^2 \to 0$ in the $\rho_\infty \to \infty$ limit, the boundary action S_c does not change the equation of motion and the boundary condition. This simply means that the mass of meson field is a physical quantity and does not depend on a regularization scheme. The regular solution is only allowed for discrete mass M. For the supersymmetric case, the meson spectrum is calculated $M^2 = 4m^2(n+1)(n+2), n = 0, 1, \ldots$. We have numerically obtained how the masses of the lowest and the second KK modes ($n = 0, 1$) change as m changes for the case dS_4 radius $l_4 = 1$. We show our numerical results by dots in figure 5 with $l_4 = 1$. Using the rescaling, we can obtain the KK (meson) masses for general l_4. For a large $m \gg 1/l_4$, i.e.

[4] It is easily checked the small fluctuations along y direction satisfy the same equations of motion.

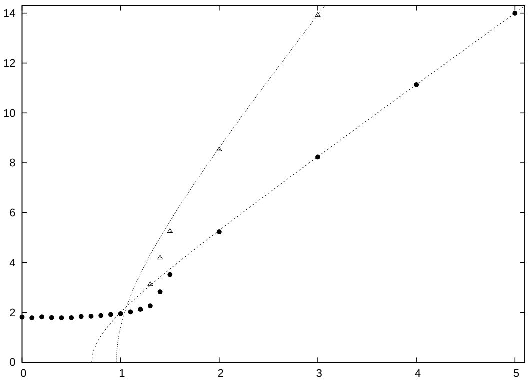

Figure 5: The lowest and second KK (meson) masses M vs quark mass m. The asymptotic forms $M = \sqrt{8m^2 - 4/l_4^2}$ and $M = \sqrt{24m^2 - 22/l_4^2}$ are given as lines. $l_4 = 1$.

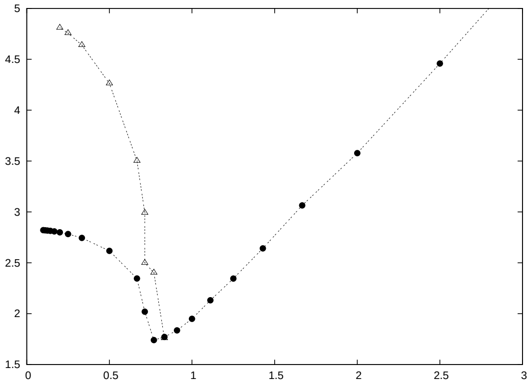

Figure 6: The first and second KK (meson) masses M vs temperature $1/l_4$ with fixing quark mass $m = 1$. (This is obtained from figure 5 using the rescaling (2.11).)

a low temperature, we find the asymptotic form for the mass of lowest KK mode

$$M^2 = 8m^2 - 4l_4^{-2} \tag{3.5}$$

which we put as a line in figure 5. Similarly the mass of second KK mode is given $M^2 = 24m^2 - 22l_4^{-2}$. The negative sign of finite temperature effect in this region is consistent with the results of QCD at finite temperature [20]. This is much clear in figure 6 where we show the temperature dependence of meson masses which is obtained from figure 5 using the rescaling (2.11) and the decreasing of meson masses in a low temperature region is due to the negative sign. This figure is an analog of [20, figure 4.4] and as long as qualitatively they behave similarly. Although the field theory in our case is different from QCD, this similarity supports that the dual field theory in our embedding of D7 brane is a field theory on dS_4 space.

The mass starts deviating from this asymptotic forms when m becomes equals to or smaller than $1.4/l_4$ where a D7-brane ends at the horizon, and the KK modes degenerate below $(1.2 \sim 1.4)/l_4$[5] in our numerical analysis. This again might signal the same phase transition in QCD at finite temperature studied in [18]. Although in this region we have numerical errors $0.1 \sim 1$ in the lowest KK mass square, the mass square never crosses zero. This is supported from the fact that the AdS horizon is stable. The flatness of the mass is understood as the large part of mass comes from thermodynamical fluctuations when the temperature $T = 2\pi/l_4$ becomes larger than the bare quark mass m. This behaviour is clear in figure 6 in which the mass increases linearly as the temperature increases in a high temperature region. These all behaviours are very similar to those in a large N gauge theory with flavor at finite temperature [21, 18].

In this section we did not study fluctuations of gauge fields on a D7-brane which correspond to vector mesons in the gauge theory side. It is known that there is a mass gap

[5] In figure 5 the degeneracy starts at $1.2/l_4$, but there are numerical errors.

in supersymmetric case [25] and thus we expect a mass gap in our system at least when $m \gg 1/l_4$. It is nontrivial whether we have tachyonic modes in vector mesons when m becomes smaller. We leave this question as a future work.

4. Wilson loops

In previous sections, we have studied the chiral condensate and meson spectrum and have obtained consistent results with those of a large N gauge theory with flavor at a finite temperature. In this section we study Wilson loops and the static quark anti-quark potential. In the AdS dual picture, the Wilson loop is described by a string lying along a geodesic in the AdS bulk with the endpoints on the boundary or a D7-brane [26, 25], which in our situation is the dS_4 hypersurface. The end points represent the positions of quark and anti-quark on a D7-brane. The Nambu-Goto action becomes

$$S = \frac{T\sqrt{-g_{tt}}}{2\pi} \int dx \sqrt{(u^2 - l_4^{-2})^2 g_{xx} + (\partial_x u)^2} \tag{4.1}$$

where we have used the coordinate system in (2.2) and u is a function of x, i.e. $u = u(x)$. g_{tt} and g_{xx} are (t,t) and (x,x) components of 4d de-Sitter metric. We assume the time scale T is much smaller than l_4 and then we approximate g_{tt} and g_{xx} are constant ($g_{xx} = 1$). In this paper we only consider the case $m \gg 1/l_4$ and further approximate the D7-brane configuration is straight $y(\rho) = m$. The open string which stretches in the AdS bulk with endpoints at $u = m$ conserves the following quantity c,

$$\frac{(u^2 - l_4^{-2})^2}{\sqrt{(u^2 - l_4^{-2})^2 + (\partial_x u)^2}} = c = u_c^2 - l_4^{-2} \tag{4.2}$$

and $\sqrt{c}$ measures the distance between the horizon $u = 1/l_4$ and the point u_c until where a string extends from $u = m$. Thus when c goes to zero, the quark and anti-quark are no longer connected by a string and move freely. The distance L between two heavy quarks is obtained from the above equation,

$$L = \frac{2}{\sqrt{c}} \int_{u_c/\sqrt{c}}^{m/\sqrt{c}} dy \frac{1}{(y^2 - c^{-1} l_4^{-2})\sqrt{(y^2 - c^{-1} l_4^{-2})^2 - 1}}. \tag{4.3}$$

The energy of Wilson loop is calculated from the on-shell Nambu-Goto action,

$$E = 2\sqrt{c} \int_{u_c/\sqrt{c}}^{m/\sqrt{c}} dy \frac{(y^2 - c^{-1} l_4^{-2})}{\sqrt{(y^2 - c^{-1} l_4^{-2})^2 - 1}}. \tag{4.4}$$

Here we have dropped various numerical factors which are not important in our discussions. It is easily seen that L (E) is given by a function of $c l_4^2$ times $c^{-1/2}$ ($c^{1/2}$). Therefore as before we can set $l_4 = 1$ without losing generality.

We divide L into three regions; (1) the short distance region $L \ll 1/m$, (2) the intermediate distance region $1/m \ll L \lesssim l_4$ and (3) the large distance region $L \gtrsim l_4$.

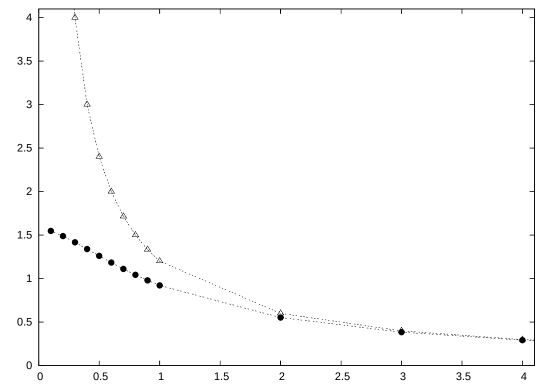

Figure 7: The distance L vs $\sqrt{c}$. The upper (bottom) curve is for $l_4 \to \infty$ ($l_4 = 1$). $m = 20$.

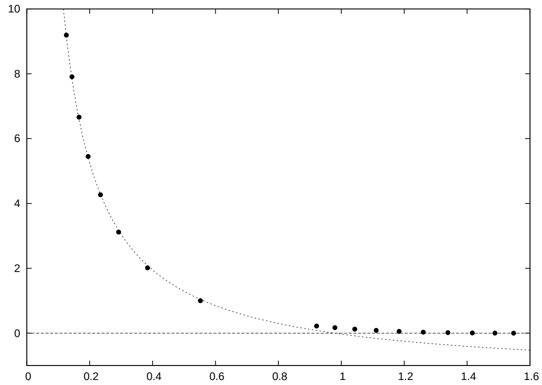

Figure 8: The binding energy $E - 2m_q$ vs L. The line is $1.32/L - 1.35/l_4$. E exactly goes to zero at $L = L_{max}$. $m = 20$. $l_4 = 1$.

We compute L and E numerically for (2) and (3) regions and show those results by dots in figure 7 and figure 8 where we put $l_4 = 1$ and took $m = 20$. In figure 7, there are two lines with dots. The upper curve is the distance L for the supersymmetric case ($l_4 \to \infty$) and the lower curve is for $l_4 = 1$. In the region (2) ($L \lesssim l_4$), they approach with each other, and the binding energy of quark and anti-quark bound state $E - 2m_q$ is well fit[6] with a Coulomb type potential

$$E - 2m_q = \frac{1.3}{L} - \frac{1.3}{l_4}, \qquad (m_q = m - l_4^{-1}) \tag{4.5}$$

which is given as a line in figure 8. The quark mass is identified as $m - l_4^{-1}$ since a string can extend up to $u = l_4^{-1}$ not $u = 0$ [27]. The difference between m_q and m obtained from the AdS/CFT dictionary [4] could be understood as a renormalization group effect since the endpoints are located at $\rho = 0$ on a D7-brane, not at $\rho = \infty$ and we should use m at a short distance region, i.e. $L \ll 1/m$. The dS_4 curvature correction $-1.3/l_4$ reduces the binding energy which is consistently understood as the finite temperature effect.

We find discrepancies from N=2 SYM when c goes to zero (region (3)). The distance L has the maximal length L_{max} for dS_4 case

$$L_{max} \sim 1.6 \times l_4\,. \tag{4.6}$$

The binding energy goes to zero at L_{max} (faster than a Coulomb type potential) from the above (figure 8) and they become free particles when L is larger than L_{max}. A similar result was obtained from AdS black holes at finite temperature [27].

We compute E and L in the $L \to 0$ limit (region (1)) and obtain

$$E = (m^2 - l_4^{-2})L\,. \tag{4.7}$$

This is the linear potential and the finite temperature effect appears as reducing the string tension as expected. (Here we chose the bare mass m as a quark mass, since $L \to 0$ is the high energy limit.)

[6] We use the data at $0.1 \leq L \leq 0.4$.

5. Conformal transformation

With the success of producing finite temperature effects from our D7-brane embedding, we would like to try answering why we have obtained such reasonable results. As briefly discussed in Introduction, the different slice of AdS space give different results on the CFT side.

We study a bulk scalar field with mass M.

$$S = \int d^5x\sqrt{-g_{AdS}}\left(-(\partial\phi)^2 - M^2\phi^2\right). \tag{5.1}$$

Since the components of two metrics (2.1) and (2.2) are same at the leading order near the AdS boundary, the asymptotic solutions of static configuration along dS_4 is given

$$\phi(u) = c_1 u^{\alpha_+} + c_2 u^{\alpha_-}, \qquad \alpha_\pm = -2 \pm \sqrt{4 + M^2 l_5^2}, \tag{5.2}$$

where c_1 and c_2 are integration constants. Here we only discuss the case $-4 \leq M^2 l_5^2 < -3$ otherwise the first correction to $c_1 u^{\alpha_+}$ asymptotic solution is larger than $c_1 u^{\alpha_-}$. (But the essential arguments below are still same for general M.) In terms of 4d Minkowski coordinate system, $u = Rt/l_4$ (appendix A),

$$\phi(u) = c_1(Rt/l_4)^{\alpha_+} + c_2(Rt/l_4)^{\alpha_-}. \tag{5.3}$$

Using the original AdS/CFT correspondence, the scaling dimension of corresponding CFT operator $\mathcal{O}(x)$ is read from this solution and is $4 + \alpha_+$. We introduce the cutoff at a constant $R = R_\infty$ and use the AdS/CFT dictionary. From straightforward calculations, we obtain the time dependent source and vacuum expectation value,

$$\mathcal{L} = \mathcal{L}_{M4} + c_1(t/l_4)^{\alpha_+}\mathcal{O}(x), \tag{5.4}$$

$$\langle\mathcal{O}(x)\rangle \propto \lim_{R_\infty\to\infty} \left.\frac{\delta S}{(Rt/l_4)^{\alpha_+}\delta c_1}\right|_{c_1\to 0} \propto c_2(t/l_4)^{\alpha_-}, \tag{5.5}$$

where $\mathcal{L}_{M4}$ is the CFT action on 4d Minkowski space, $x = (t, x_i)$. (Notice that the on-shell action from $t \to \pm\infty$ goes to zero compared to that from the cut $R = R_\infty$ as the cut approaches to the AdS boundary.) From the D3-brane effective theory's point of view on Minkowski space, we are considering responses against time dependent sources. As discussed in section 2, the scalar field on D7-brane corresponds to the quark anti-quark composite operator. Therefore we were studying a time dependent mass term for quarks (thus supersymmetry is broken) and responses to that.

On the other hand, the (A)dS/dS correspondence claims that introducing the cutoff at constant $u = u_\infty$ and applying the AdS/CFT dictionary. We then obtain

$$\mathcal{L} = \mathcal{L}_{dS4} + c_1\mathcal{O}(x), \tag{5.6}$$

$$\langle\mathcal{O}(x)\rangle \propto c_2, \tag{5.7}$$

where $\mathcal{L}_{dS4}$ is the CFT action on dS_4, $x = (s, x_i)$ (appendix A), and the vacuum expectation value is calculated around dS_4 background. The operator $\mathcal{O}$ is same since the asymptotic solution is same.

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The cut near the AdS boundary corresponds to the UV cutoff in the CFT side. The above result indicates a different UV regularization induces two different results on correlation functions and the difference does not seem due to just a different scheme. Let us try to give some explanation on this point. We again study the on-shell value of action. If we keep c_1 nonzero and compute the on-shell action, we obtain in each case

$$S = \int d^4x\sqrt{-g_{M4}}\frac{R^5}{l_5^5}\phi\partial_R\phi\Big|^{R=R_\infty} = \int d^4x\sqrt{-g_{M4}}\frac{\alpha_+c_1^2t^{2\alpha_+}}{l_5^5l_4^{2\alpha_+}}R_\infty^{4+2\alpha_+}+\cdots, \tag{5.8}$$

$$S = \int d^4x\sqrt{-g_{dS4}}\left(\frac{u^2}{l_5^2}-\frac{l_5^2}{l_4^2}\right)^{5/2}\phi\partial_u\phi\Big|^{u=u_\infty} = \int d^4x\sqrt{-g_{dS4}}\frac{\alpha_+c_1^2}{l_5^5}u_\infty^{4+2\alpha_+}+\cdots. \tag{5.9}$$

(Here we ignored the possible contributions from $t=\pm\infty$.) The leading contribution is divergent and takes a different value except when $t\sim s\sim l_4$. Thus they are not related just by a finite renormalization.

Because of a time dependent coupling, we have time dependent divergences (5.8). Thus we may mix time t and the UV cutoff R_∞ and define a new time s, and similarly define a new UV cutoff u_∞ such that the time dependence disappears. (This is nothing but a coordinate transformation from AdS point of view (A.10).) Taking the limit $R_\infty\to\infty$, the mixture becomes zero and the new time s is still t (and so the metric unchanged). Since dS_4 is conformal to flat, we act the conformal transformation (and scale transformation to fields) and change the metric from Minkowski to dS_4,

$$g_{M4} = \Omega^{-2}g_{dS4}, \qquad \Omega = \frac{l_4}{s} = \frac{l_4}{t}. \tag{5.10}$$

Since N=4 SYM, $\mathcal{L}_{M4}(\Phi)$ (Φ represents the fields in the CFT), is scale invariant, the resultant action $\mathcal{L}_{dS4}$ becomes $\mathcal{L}_{dS4}=\mathcal{L}_{M4}(\Phi)+\mathcal{L}(\partial\Omega,\Omega,\Phi)$ and the contraction of Lorentz indices is given by dS_4 metric. (We do not consider anomaly here.) The second part is the terms which always include the (time) derivative of Ω[7]. Since the scaling dimension of $\mathcal{O}$ is $4+\alpha_+$, $c_1(t/l_4)^{\alpha_+}\mathcal{O}(x)$ part in Lagrangian is transformed into $c_1\mathcal{O}(x)$ and the vacuum expectation value $\langle\mathcal{O}(x)\rangle\propto c_2(t/l_4)^{\alpha_-}$ is also transformed into $\langle\mathcal{O}(x)\rangle\propto c_2$. Although this argument is quite unsatisfactory, the coupling constant and vacuum expectation value in CFT on Minkowski is conformally transformed into those in CFT on de-Sitter space. The cutoff on Minkowski space is also mapped into the cutoff on de-Sitter space. We hope this is touching something behind the (A)dS/dS correspondence and why we obtained reasonable results.

6. Conclusions and discussions

We have studied a D7-brane probe in $AdS_5\times S^5$ geometry. We identify the normal direction to a dS_4 hypersurface in AdS_5 as the energy scale in the dual gauge theory and obtain a D7-brane configuration which is static along the dS_4 directions. The scalar field on a

[7] The D3-brane action might have couplings to gravity, when gravity is coupled, which are consistent with superconformal invariance, such as $R\phi^2/6$ for a scalar field ϕ and R is the scalar curvature of the background 4d metric [28]. Then the second part is written with a background gravity in a general covariant way.

D7-brane is a bulk field of AdS_5 foliated by dS_4 and can be treated as a bulk field in the Randall Sundrum model with non fine-tuned brane(s) or in the (A)dS/dS correspondence studied by Alishahiha et al. [12, 13]. Thus the field theory dual is expected to be a CFT with flavor on dS_4. We have checked this expectation is true by identifying the finite temperature effects in the chiral condensate, meson mass spectrum and quark anti-quark potential and showing they are properly reproduced. Therefore we realize the gravity dual of non supersymmetric field theory by breaking supersymmetry by a probe brane.

The regularized on-shell action is log divergent in terms of m, which is the asymptotic locus of the D7-brane, and thus there is an infinite potential barrier between the dS embedding and supersymmetric embedding. This might indicate that different UV regularizations induce renormalization flows into different IR physics which are not connected by perturbative modes or finite renormalizations, but might be transformed with each other by a complicated (conformal) transformation.

We showed there is no tachyonic mode in the perturbations of scalar fields localized on the D7-brane. We expect there is no tachyonic mode in the perturbations of gauge fields on the D7-brane as well, since there is a mass gap in the supersymmetric limit. When we include the back reaction of D7-brane, the gravitational solution is expected to be still similar. Thus this realization of de-Sitter space on the D7-brane might be phenomenologically interesting, since the possibility of the localized gravity in D3/D7 system is discussed in [29].

We have little knowledge on the quantum field theory on de-Sitter space. Because of numerical error, we did not pay much attention to the high temperature regions. It might be interesting to study them and phase transitions by improving the accuracy of numerical analysis. It might be also interesting to study two point or multi point functions in this direction.

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A. Minkowski and de-Sitter slices in Anti de-Sitter space

The AdS_5 with the curvature radius l_5 is a hypersurface

$$X_0^2 + X_5^2 - X_i^2 - X_4^2 = l_5^2, \quad i = 1, 2, 3, \tag{A.1}$$

in the 6 dimensional flat space with the metric

$$ds^2 = -dX_5^2 + dX_i^2 + d(X_4 + X_0)d(X_4 - X_0). \tag{A.2}$$

The 4d Minkowski space with coordinates (t, x_i) is embedded as follows,

$$X_5 = \frac{Rt}{l_5}, \quad X_i = \frac{Rx_i}{l_5}, \quad X_4 - X_0 = -R, \quad X_4 + X_0 = \frac{l_5^2}{R} + \frac{R}{l_5^2}(\vec{x}^2 - t^2), \tag{A.3}$$

and the AdS_5 metric is given

$$ds^2 = \frac{R^2}{l_5^2}(-dt^2 + d\vec{x}^2) + \frac{l_5^2}{R^2}dR^2. \tag{A.4}$$

Similarly the dS_4 space with coordinates (s, x_i) is embedded along X_0, X_i and X_4 satisfying

$$X_0^2 - X_i^2 - X_4^2 = -l_4^2\left(\frac{u^2}{l_5^2} - \frac{l_5^2}{l_4^2}\right). \tag{A.5}$$

The embedding is

$$X_5 = \frac{l_4}{l_5}u, \quad X_i = \frac{l_4 x_i}{s}\sqrt{\frac{u^2}{l_5^2} - \frac{l_5^2}{l_4^2}}, \quad X_4 - X_0 = \frac{l_4}{s}\sqrt{\frac{u^2}{l_5^2} - \frac{l_5^2}{l_4^2}}, \tag{A.6}$$

$$X_4 + X_0 = l_4\left(s - \frac{\vec{x}^2}{s}\right)\sqrt{\frac{u^2}{l_5^2} - \frac{l_5^2}{l_4^2}}, \tag{A.7}$$

and AdS_5 metric becomes

$$ds^2 = \left(\frac{u^2}{l_5^2} - \frac{l_5^2}{l_4^2}\right)\left(\frac{l_4^2}{s^2}(-ds^2 + d\vec{x}^2)\right) + \left(\frac{u^2}{l_5^2} - \frac{l_5^2}{l_4^2}\right)^{-1} du^2. \tag{A.8}$$

The transformation between two coordinate systems is easily obtained from the embeddings:

$$x_i = x_i \tag{A.9}$$

$$t = \frac{su}{l_5}\left(\frac{u^2}{l_5^2} - \frac{l_5^2}{l_4^2}\right)^{-1/2} \leftrightarrow s = \left(t^2 - \frac{l_5^4}{R^2}\right)^{1/2} \tag{A.10}$$

$$R = \frac{l_4 l_5}{s}\left(\frac{u^2}{l_5^2} - \frac{l_5^2}{l_4^2}\right)^{1/2} \leftrightarrow u = \frac{Rt}{l_4} \tag{A.11}$$

The AdS boundary corresponds to $R \to \infty$ and equivalently to $u \to \infty$. Near the boundary, we have $t = s$.

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